

Transformational Geometry

MODULE 9

Transformations and Congruence

FL 8.G.1.1, 8.G.1.2, 8.G.1.3

MODULE 10

Transformations and Similarity

FL 8.G.1.3, 8.G.1.4

CAREERS IN MATH

Contractor A contractor is engaged in the construction, repair, and dismantling of structures such as buildings, bridges, and roads. Contractors use math when researching and implementing building codes, making measurements and scaling models, and in financial management.

If you are interested in a career as a contractor, you should study the following mathematical subjects:

- Business Math
- Geometry
- Algebra
- Trigonometry

Research other careers that require the use of business math and scaling.

Unit 4 Performance Task

At the end of the unit, check out how **contractors** use math.

Vocabulary Preview

Use the puzzle to preview key vocabulary from this unit. Unscramble the circled letters within found words to answer the riddle at the bottom of the page.

W	T	C	F	V	A	F	I	L	I	T	U	G	N	S
M	U	J	L	O	C	Z	H	B	R	S	D	E	O	W
F	E	Q	V	H	B	T	E	A	N	T	P	X	I	A
M	Y	J	G	B	O	G	N	P	K	G	G	B	T	Q
N	X	L	H	A	A	S	E	V	Z	R	U	B	C	W
L	O	X	D	M	L	A	G	R	H	H	L	R	U	A
G	P	I	I	A	E	M	X	K	V	A	K	X	D	K
B	D	E	T	N	E	M	E	G	R	A	L	N	E	A
W	R	I	E	C	A	N	R	E	Y	M	A	T	R	P
P	O	G	L	K	E	R	O	T	A	T	I	O	N	R
N	Z	Y	E	A	C	L	R	O	V	Z	S	P	I	U
U	Y	T	J	N	T	Q	F	N	B	X	G	G	C	J
O	N	E	R	Z	I	I	P	E	I	Y	F	A	B	W
I	F	V	U	C	M	S	O	G	R	P	Q	K	B	W
C	Q	T	U	I	C	C	U	N	L	N	T	L	D	Y

The input of a transformation. (Lesson 9.1)

A transformation that flips a figure across a line. (Lesson 9.2)

A transformation that slides a figure along a straight line. (Lesson 9.1)

A transformation that turns a figure around a given point. (Lesson 9.3)

The product of a figure made larger by dilation. (Lesson 10.1)

The product of a figure made smaller by dilation. (Lesson 10.1)

Scaled replicas that change the size but not the shape of a figure. (Lesson 10.1)

Q: What do you call an angle that's broken?

A: A _____!

Transformations and Congruence



ESSENTIAL QUESTION

How can you use transformations and congruence to solve real-world problems?

MODULE



9

LESSON 9.1

Properties of Translations



FL 8.G.1.1, 8.G.1.3

LESSON 9.2

Properties of Reflections



FL 8.G.1.1, 8.G.1.3

LESSON 9.3

Properties of Rotations



FL 8.G.1.1, 8.G.1.3

LESSON 9.4

Algebraic Representations of Transformations



FL 8.G.1.3

LESSON 9.5

Congruent Figures



FL 8.G.1.2



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Real-World Video

When a marching band lines up and marches across the field, they are modeling a translation. As they march, they maintain size and orientation. A translation is one type of transformation.

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Animated Math

Interactively explore key concepts to see how math works.

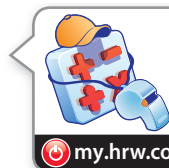


Personal Math Trainer

Get immediate feedback and help as you work through practice sets.

Are YOU Ready?

Complete these exercises to review skills you will need for this module.



**Personal
Math Trainer**

Online
Assessment and
Intervention

Integer Operations

EXAMPLE $-3 - (-6) = -3 + 6$
 $= |-3| - |6|$
 $= 3$

To subtract an integer, add its opposite.
 The signs are different, so find the
 difference of the absolute values: $6 - 3 = 3$.
 Use the sign of the number with the greater
 absolute value.

Find each difference.

1. $5 - (-9)$

2. $-6 - 8$

3. $2 - 9$

4. $-10 - (-6)$

5. $3 - (-11)$

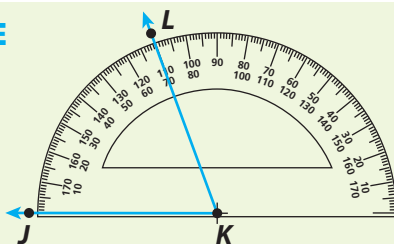
6. $12 - 7$

7. $-4 - 11$

8. $0 - (-12)$

Measure Angles

EXAMPLE

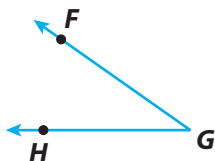


$m\angle JKL = 70^\circ$

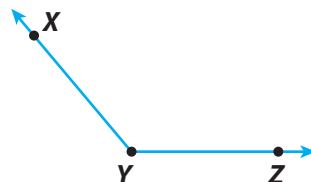
Place the center point of the
 protractor on the angle's vertex.
 Align one ray with the base of the
 protractor.
 Read the angle measure where the
 other ray intersects the semicircle.

Use a protractor to measure each angle.

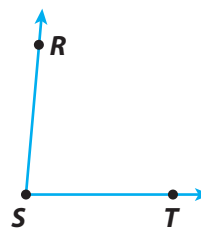
9.



10.



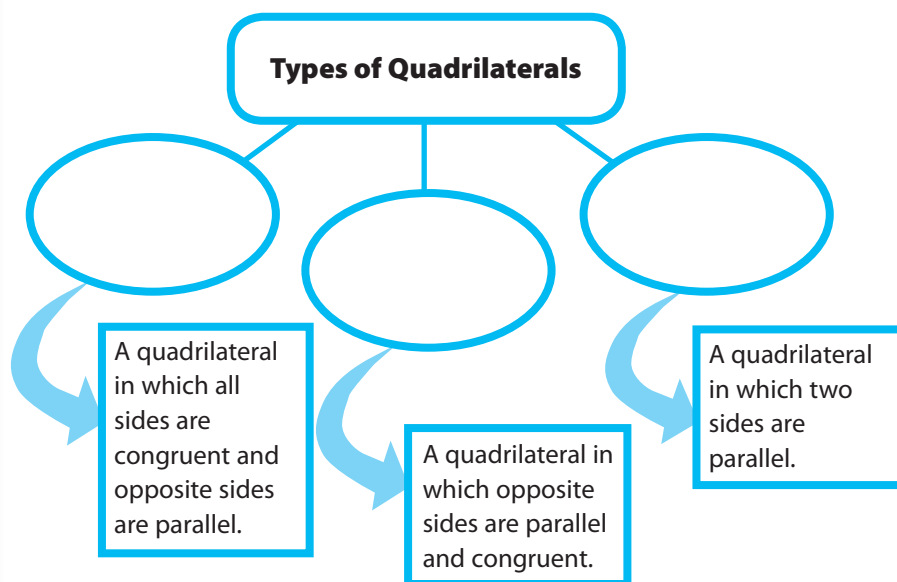
11.



Reading Start-Up

Visualize Vocabulary

Use the ✓ words to complete the graphic organizer. You will put one word in each oval.



Understand Vocabulary

Match the term on the left to the correct expression on the right.

- | | |
|-------------------|--|
| 1. transformation | A. A function that describes a change in the position, size, or shape of a figure. |
| 2. reflection | B. A function that slides a figure along a straight line. |
| 3. translation | C. A transformation that flips a figure across a line. |

Vocabulary

Review Words

- coordinate plane (*plano cartesiano*)
- ✓ parallelogram (*paralelogramo*)
- quadrilateral (*cuadrilátero*)
- ✓ rhombus (*rombo*)
- ✓ trapezoid (*trapecio*)

Preview Words

- center of rotation (*centro de rotación*)
- congruent (*congruente*)
- image (*imagen*)
- line of reflection (*línea de reflexión*)
- preimage (*imagen original*)
- reflection (*reflexión*)
- rotation (*rotación*)
- transformation (*transformación*)
- translation (*traslación*)

Active Reading

Booklet Before beginning the module, create a booklet to help you learn the concepts in this module. Write the main idea of each lesson on each page of the booklet. As you study each lesson, write important details that support the main idea, such as vocabulary and formulas. Refer to your finished booklet as you work on assignments and study for tests.





MODULE 9

Unpacking the Standards

Understanding the standards and the vocabulary terms in the standards will help you know exactly what you are expected to learn in this module.



FL 8.G.1.2

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

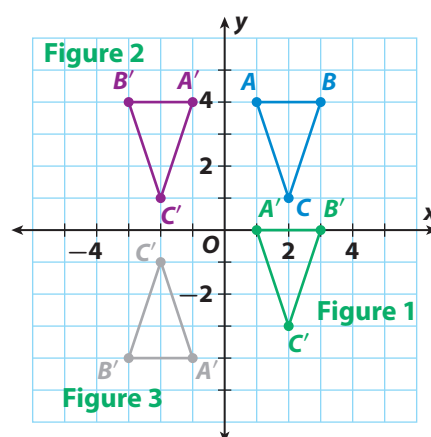
What It Means to You

You will identify a rotation, a reflection, a translation, and a sequence of transformations, and understand that the image has the same shape and size as the preimage.

UNPACKING EXAMPLE 8.G.1.2

The figure shows triangle ABC and its image after three different transformations. Identify and describe the translation, the reflection, and the rotation of triangle ABC .

Figure 1 is a translation 4 units down. Figure 2 is a reflection across the y -axis. Figure 3 is a rotation of 180° .



FL 8.G.1.3

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

What It Means to You

You can use an algebraic representation to translate, reflect, or rotate a two-dimensional figure.

UNPACKING EXAMPLE 8.G.1.3

Rectangle $RSTU$ with vertices $(-4, 1)$, $(-1, 1)$, $(-1, -3)$, and $(-4, -3)$ is reflected across the y -axis. Find the coordinates of the image.

The rule to reflect across the y -axis is to change the sign of the x -coordinate.

Coordinates	Reflect across the y -axis $(-x, y)$	Coordinates of image
$(-4, 1)$, $(-1, 1)$,	$(-(-4), 1)$, $(-(-1), 1)$,	$(4, 1)$, $(1, 1)$,
$(-1, -3)$, $(-4, -3)$	$(-(-1), -3)$, $(-(-4), -3)$	$(1, -3)$, $(4, -3)$

The coordinates of the image are $(4, 1)$, $(1, 1)$, $(1, -3)$, and $(4, -3)$.



Visit my.hrw.com to see all **Florida Math Standards** unpacked.

LESSON 9.1 Properties of Translations



FL 8.G.1.1

Verify experimentally the properties of ... translations. Also 8.G.1.1a, 8.G.1.1b, 8.G.1.1c, 8.G.1.3



ESSENTIAL QUESTION

How do you describe the properties of translation and their effect on the congruence and orientation of figures?

EXPLORE ACTIVITY 1



FL 8.G.1.1

Exploring Translations

You learned that a function is a rule that assigns exactly one output to each input. A **transformation** is a function that describes a change in the position, size, or shape of a figure. The input of a transformation is the **preimage**, and the output of a transformation is the **image**.

A **translation** is a transformation that slides a figure along a straight line.

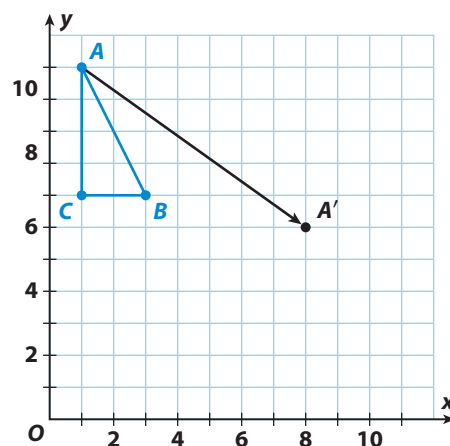
The triangle shown on the grid is the **preimage (input)**. The arrow shows the motion of a translation and how point **A** is translated to point **A'**.

- Trace triangle ABC onto a piece of paper. Cut out your traced triangle.
- Slide your triangle along the arrow to model the translation that maps point A to point A' .
- The image of the translation is the triangle produced by the translation. Sketch the image of the translation.
- The vertices of the image are labeled using prime notation. For example, the image of A is A' . Label the images of points B and C .

- Describe the motion modeled by the translation.

Move _____ units right and _____ units down.

- Check that the motion you described in part **E** is the same motion that maps point A onto A' , point B onto B' , and point C onto C' .



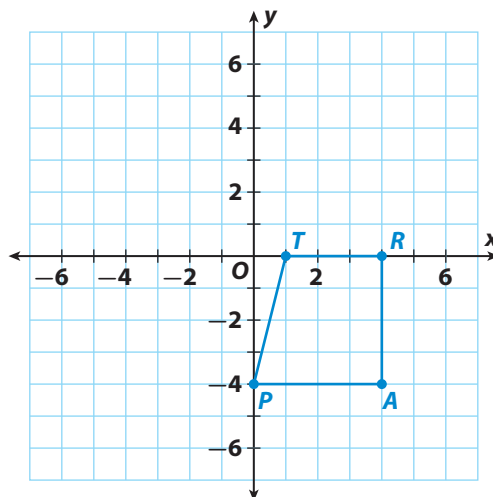
Reflect

- How is the orientation of the triangle affected by the translation?



Properties of Translations

Use trapezoid $TRAP$ to investigate the properties of translations.



- A** Trace the trapezoid onto a piece of paper. Cut out your traced trapezoid.
- B** Place your trapezoid on top of the trapezoid in the figure. Then translate your trapezoid 5 units to the left and 3 units up. Sketch the image of the translation by tracing your trapezoid in this new location. Label the vertices of the image T' , R' , A' , and P' .
- C** Use a ruler to measure the sides of trapezoid $TRAP$ in centimeters.

$TR =$ _____ $RA =$ _____ $AP =$ _____ $TP =$ _____

- D** Use a ruler to measure the sides of trapezoid $T'R'A'P'$ in centimeters.

$T'R' =$ _____ $R'A' =$ _____ $A'P' =$ _____ $T'P' =$ _____

- E** What do you notice about the lengths of corresponding sides of the two figures?

- F** Use a protractor to measure the angles of trapezoid $TRAP$.

$m\angle T =$ _____ $m\angle R =$ _____ $m\angle A =$ _____ $m\angle P =$ _____

- G** Use a protractor to measure the angles of trapezoid $T'R'A'P'$.

$m\angle T' =$ _____ $m\angle R' =$ _____ $m\angle A' =$ _____ $m\angle P' =$ _____

- H** What do you notice about the measures of corresponding angles of the two figures?

- I** Which sides of trapezoid $TRAP$ are parallel? How do you know?

Which sides of trapezoid $T'R'A'P'$ are parallel? _____

What do you notice? _____

Reflect

2. **Make a Conjecture** Use your results from parts **E**, **H**, and **I** to make a conjecture about translations.

3. Two figures that have the same size and shape are called *congruent*. What can you say about translations and congruence?

Graphing Translations

To translate a figure in the coordinate plane, translate each of its vertices. Then connect the vertices to form the image.

EXAMPLE 1



FL 8.G.1.3

The figure shows triangle XYZ . Graph the image of the triangle after a translation of 4 units to the right and 1 unit up.

STEP 1 Translate point X .

Count right 4 units and up 1 unit and plot point X' .

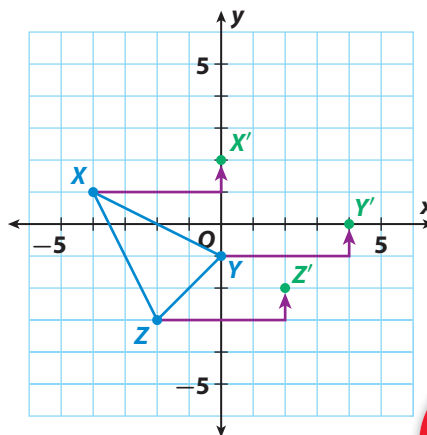
STEP 2 Translate point Y .

Count right 4 units and up 1 unit and plot point Y' .

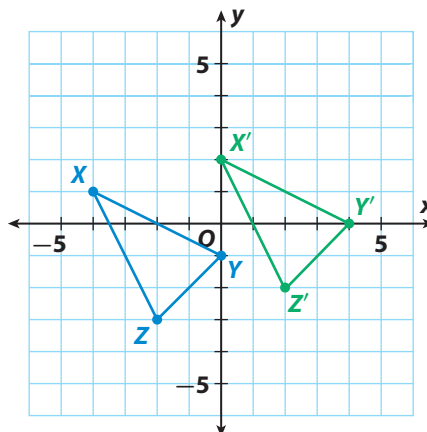
STEP 3 Translate point Z .

Count right 4 units and up 1 unit and plot point Z' .

STEP 4 Connect X' , Y' , and Z' to form triangle $X'Y'Z'$.



Each vertex is moved 4 units right and 1 unit up.



Math On the Spot

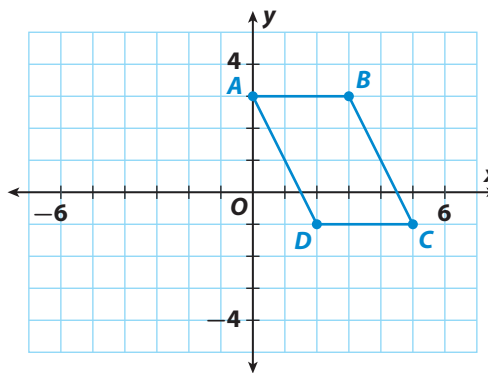
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Math Talk
Mathematical Practices

Is the image congruent to the preimage? How do you know?

YOUR TURN

4. The figure shows parallelogram $ABCD$. Graph the image of the parallelogram after a translation of 5 units to the left and 2 units down.

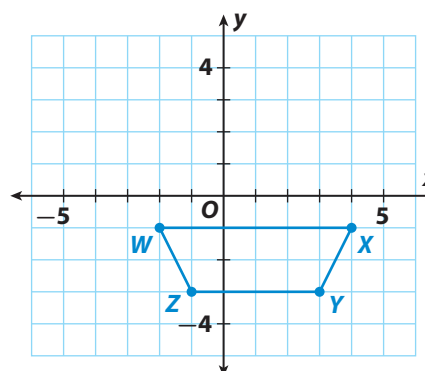


Guided Practice

- Vocabulary** A _____ is a change in the position, size, or shape of a figure.
- Vocabulary** When you perform a transformation of a figure on the coordinate plane, the input of the transformation is called the _____, and the output of the transformation is called the _____.
- Joni translates a right triangle 2 units down and 4 units to the right. How does the orientation of the image of the triangle compare with the orientation of the preimage? ([Explore Activity 1](#))

- Rashid drew rectangle $PQRS$ on a coordinate plane. He then translated the rectangle 3 units up and 3 units to the left and labeled the image $P'Q'R'S'$. How do rectangle $PQRS$ and rectangle $P'Q'R'S'$ compare? ([Explore Activity 2](#))

- The figure shows trapezoid $WXYZ$. Graph the image of the trapezoid after a translation of 4 units up and 2 units to the left. ([Example 1](#))



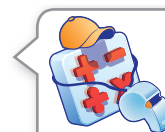
ESSENTIAL QUESTION CHECK-IN

- What are the properties of translations?

9.1 Independent Practice



FL 8.G.1.1, 8.G.1.3



Personal Math Trainer

Online Assessment and Intervention

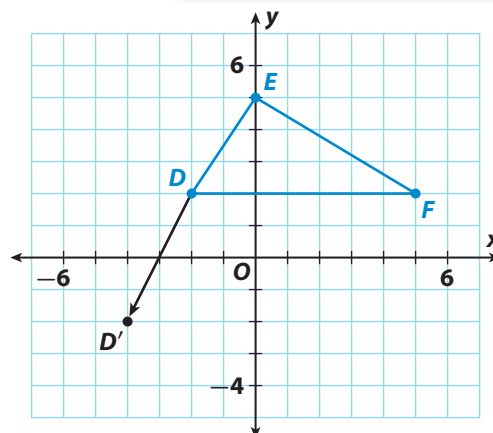
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7. The figure shows triangle DEF .

a. Graph the image of the triangle after the translation that maps point D to point D' .

b. How would you describe the translation?

c. How does the image of triangle DEF compare with the preimage?

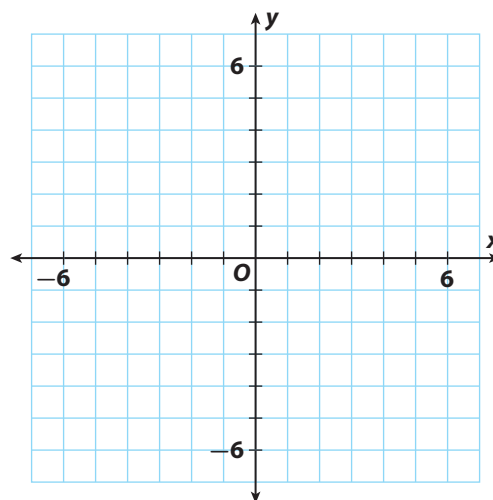


8. a. Graph quadrilateral $KLMN$ with vertices $K(-3, 2)$, $L(2, 2)$, $M(0, -3)$, and $N(-4, 0)$ on the coordinate grid.

b. On the same coordinate grid, graph the image of quadrilateral $KLMN$ after a translation of 3 units to the right and 4 units up.

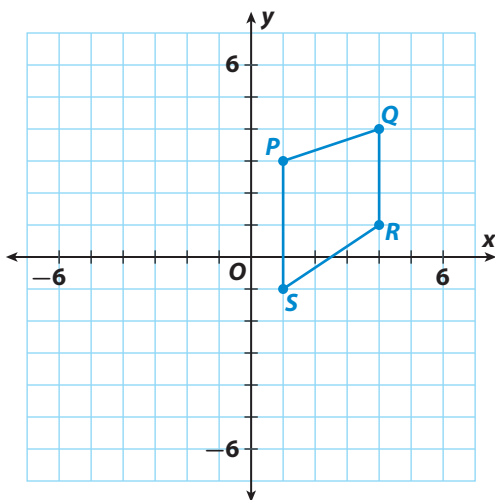
c. Which side of the image is congruent to side \overline{LM} ?

Name three other pairs of congruent sides.

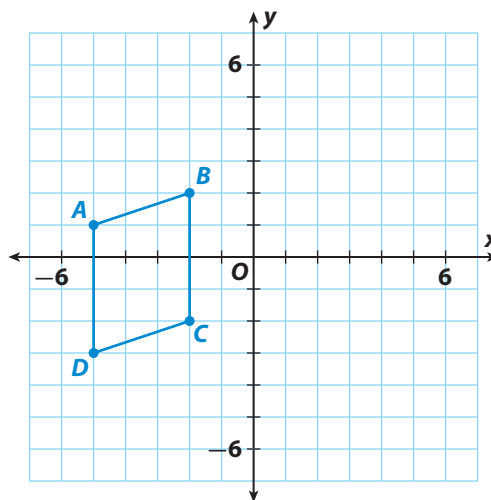


Draw the image of the figure after each translation.

9. 4 units left and 2 units down

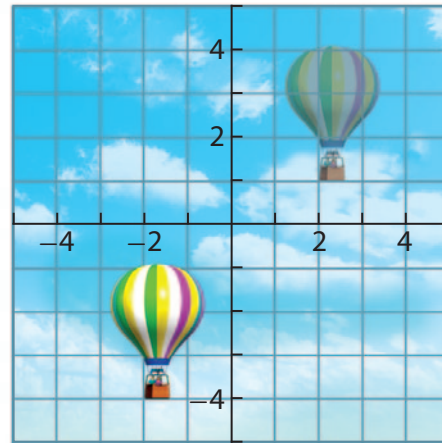


10. 5 units right and 3 units up



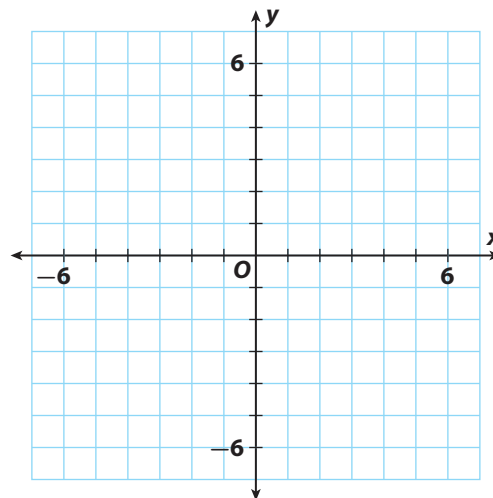
11. The figure shows the ascent of a hot air balloon. How would you describe the translation?

12. **Critical Thinking** Is it possible that the orientation of a figure could change after it is translated? Explain.



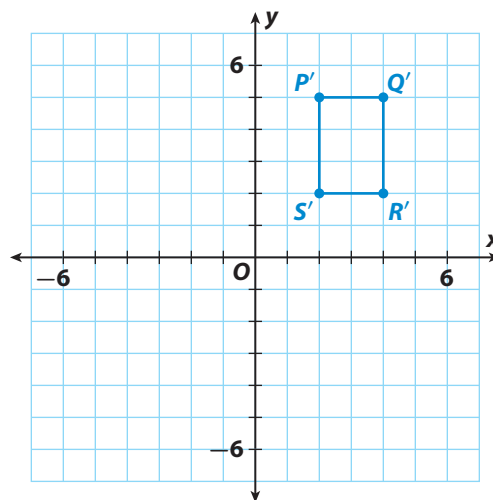
FOCUS ON HIGHER ORDER THINKING

13. a. **Multistep** Graph triangle XYZ with vertices $X(-2, -5)$, $Y(2, -2)$, and $Z(4, -4)$ on the coordinate grid.
- b. On the same coordinate grid, graph and label triangle $X'Y'Z'$, the image of triangle XYZ after a translation of 3 units to the left and 6 units up.
- c. Now graph and label triangle $X''Y''Z''$, the image of triangle $X'Y'Z'$ after a translation of 1 unit to the left and 2 units down.
- d. **Analyze Relationships** How would you describe the translation that maps triangle XYZ onto triangle $X''Y''Z''$?



14. **Critical Thinking** The figure shows rectangle $P'Q'R'S'$, the image of rectangle $PQRS$ after a translation of 5 units to the right and 7 units up. Graph and label the preimage $PQRS$.

15. **Communicate Mathematical Ideas** Explain why the image of a figure after a translation is congruent to its preimage.



LESSON 9.2 Properties of Reflections

 **FL** 8.G.1.1

Verify experimentally the properties of ... reflections.... Also 8.G.1.1a, 8.G.1.1b, 8.G.1.1c, 8.G.1.3



ESSENTIAL QUESTION

How do you describe the properties of reflection and their effect on the congruence and orientation of figures?

EXPLORE ACTIVITY 1

 **FL** 8.G.1.1

Exploring Reflections

A **reflection** is a transformation that flips a figure across a line. The line is called the **line of reflection**. Each point and its image are the same distance from the line of reflection.

The triangle shown on the grid is the preimage. You will explore reflections across the x - and y -axes.

- Trace triangle ABC and the x - and y -axes onto a piece of paper.
- Fold your paper along the x -axis and trace the image of the triangle on the opposite side of the x -axis. Unfold your paper and label the vertices of the image A' , B' , and C' .
- What is the line of reflection for this transformation?

- Find the perpendicular distance from each point to the line of reflection.

Point A _____ Point B _____ Point C _____

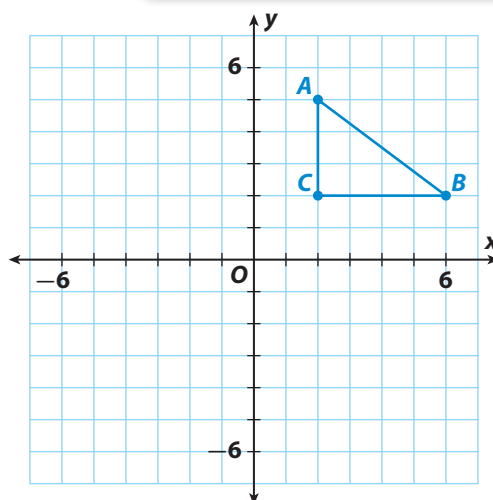
- Find the perpendicular distance from each point to the line of reflection.

Point A' _____ Point B' _____ Point C' _____

- What do you notice about the distances you found in **D** and **E**?

Reflect

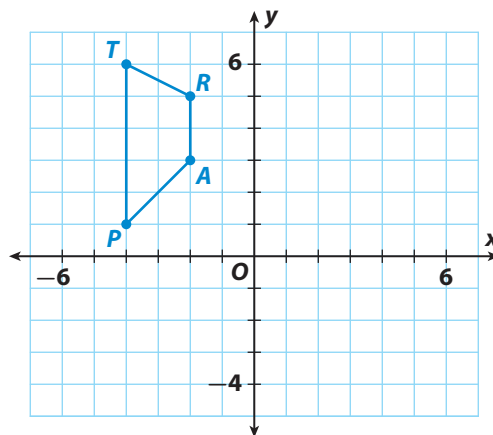
- Fold your paper from **A** along the y -axis and trace the image of triangle ABC on the opposite side. Label the vertices of the image A'' , B'' , and C'' . What is the line of reflection for this transformation?
- How does each image in your drawings compare with its preimage?





Properties of Reflections

Use trapezoid $TRAP$ to investigate the properties of reflections.



A Trace the trapezoid onto a piece of paper. Cut out your traced trapezoid.

B Place your trapezoid on top of the trapezoid in the figure. Then reflect your trapezoid across the y -axis. Sketch the image of the reflection by tracing your trapezoid in this new location. Label the vertices of the image T' , R' , A' , and P' .

C Use a ruler to measure the sides of trapezoid $TRAP$ in centimeters.

$TR = \underline{\hspace{2cm}}$ $RA = \underline{\hspace{2cm}}$ $AP = \underline{\hspace{2cm}}$ $TP = \underline{\hspace{2cm}}$

D Use a ruler to measure the sides of trapezoid $T'R'A'P'$ in centimeters.

$T'R' = \underline{\hspace{2cm}}$ $R'A' = \underline{\hspace{2cm}}$ $A'P' = \underline{\hspace{2cm}}$ $T'P' = \underline{\hspace{2cm}}$

E What do you notice about the lengths of corresponding sides of the two figures?

F Use a protractor to measure the angles of trapezoid $TRAP$.

$m\angle T = \underline{\hspace{2cm}}$ $m\angle R = \underline{\hspace{2cm}}$ $m\angle A = \underline{\hspace{2cm}}$ $m\angle P = \underline{\hspace{2cm}}$

G Use a protractor to measure the angles of trapezoid $T'R'A'P'$.

$m\angle T' = \underline{\hspace{2cm}}$ $m\angle R' = \underline{\hspace{2cm}}$ $m\angle A' = \underline{\hspace{2cm}}$ $m\angle P' = \underline{\hspace{2cm}}$

H What do you notice about the measures of corresponding angles of the two figures?

I Which sides of trapezoid $TRAP$ are parallel? _____

Which sides of trapezoid $T'R'A'P'$ are parallel? _____
What do you notice?

Reflect

3. **Make a Conjecture** Use your results from **E**, **H**, and **I** to make a conjecture about reflections.



Math Talk

Mathematical Practices

What can you say about reflections and congruence?

Graphing Reflections

To reflect a figure across a line of reflection, reflect each of its vertices. Then connect the vertices to form the image. Remember that each point and its image are the same distance from the line of reflection.

EXAMPLE 1



FL 8.G.1.3

The figure shows triangle XYZ . Graph the image of the triangle after a reflection across the x -axis.

STEP 1

Reflect point X .

Point X is 3 units below the x -axis. Count 3 units above the x -axis and plot point X' .

STEP 2

Reflect point Y .

Point Y is 1 unit below the x -axis. Count 1 unit above the x -axis and plot point Y' .

STEP 3

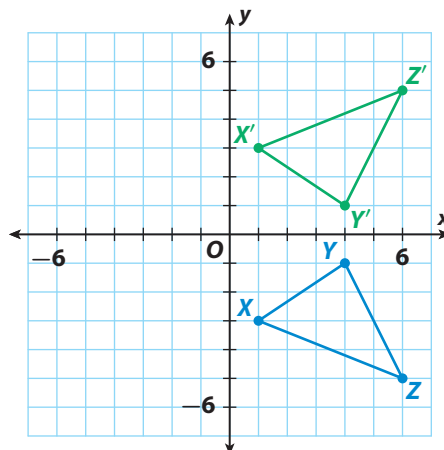
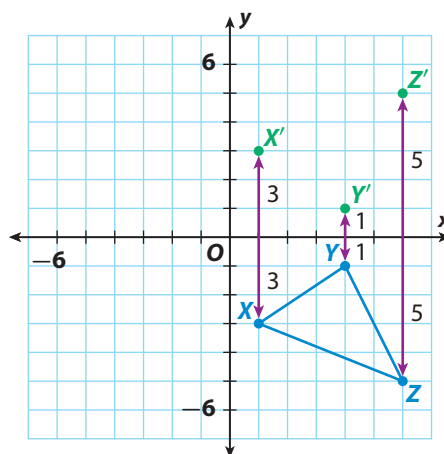
Reflect point Z .

Point Z is 5 units below the x -axis. Count 5 units above the x -axis and plot point Z' .

STEP 4

Connect X' , Y' , and Z' to form triangle $X'Y'Z'$.

Each vertex of the image is the same distance from the x -axis as the corresponding vertex in the original figure.



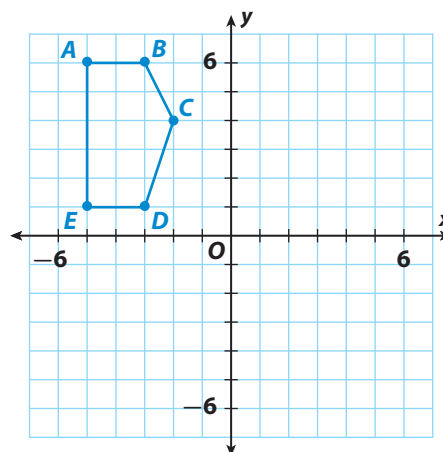
Math On the Spot

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My Notes

YOUR TURN

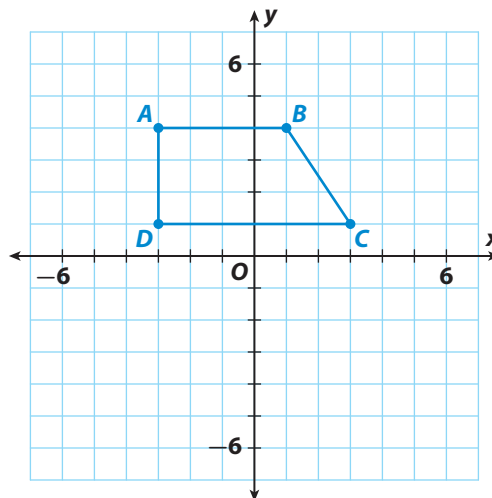
4. The figure shows pentagon $ABCDE$. Graph the image of the pentagon after a reflection across the y -axis.



Guided Practice

1. **Vocabulary** A reflection is a transformation that flips a figure across a line called the _____.
2. The figure shows trapezoid $ABCD$. (*Explore Activities 1 and 2 and Example 1*)

- a. Graph the image of the trapezoid after a reflection across the x -axis. Label the vertices of the image.
- b. How do trapezoid $ABCD$ and trapezoid $A'B'C'D'$ compare?



- c. **What If?** Suppose you reflected trapezoid $ABCD$ across the y -axis. How would the orientation of the image of the trapezoid compare with the orientation of the preimage?



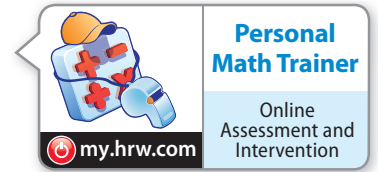
ESSENTIAL QUESTION CHECK-IN

3. What are the properties of reflections?

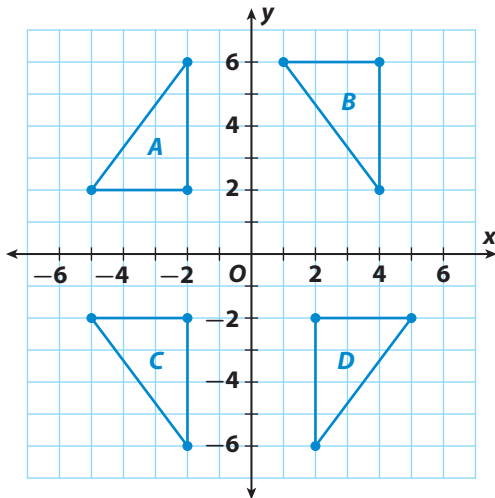
9.2 Independent Practice



FL 8.G.1.1, 8.G.1.3



The graph shows four right triangles. Use the graph for Exercises 4–7.



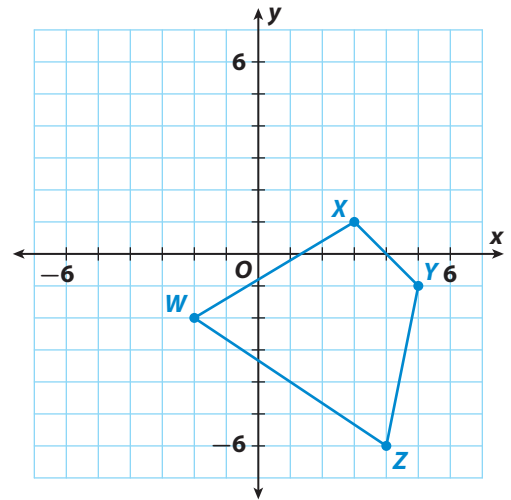
4. Which two triangles are reflections of each other across the x -axis?

5. For which two triangles is the line of reflection the y -axis?

6. Which triangle is a translation of triangle C ? How would you describe the translation?

7. Which triangles are congruent? How do you know?

8. a. Graph quadrilateral $WXYZ$ with vertices $W(-2, -2)$, $X(3, 1)$, $Y(5, -1)$, and $Z(4, -6)$ on the coordinate grid.



- b. On the same coordinate grid, graph quadrilateral $W'X'Y'Z'$, the image of quadrilateral $WXYZ$ after a reflection across the x -axis.
- c. Which side of the image is congruent to side \overline{YZ} ?

Name three other pairs of congruent sides.

- d. Which angle of the image is congruent to $\angle X$?

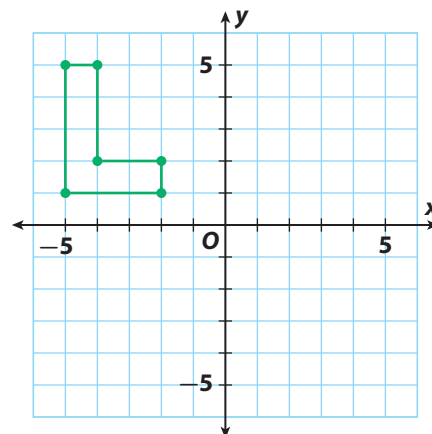
Name three other pairs of congruent angles.

9. **Critical Thinking** Is it possible that the image of a point after a reflection could be the same point as the preimage? Explain.

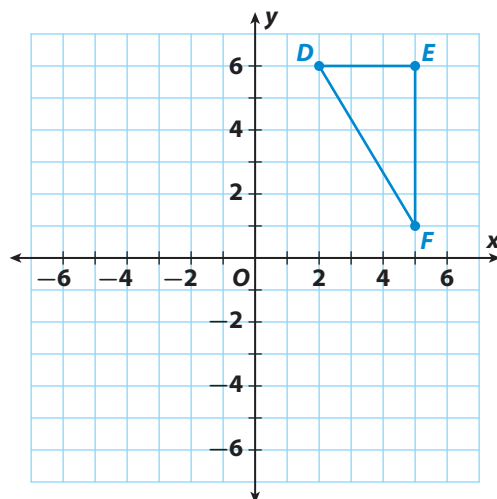


FOCUS ON HIGHER ORDER THINKING

10. a. Graph the image of the figure shown after a reflection across the y-axis.
- b. On the same coordinate grid, graph the image of the figure you drew in part a after a reflection across the x-axis.
- c. **Make a Conjecture** What other sequence of transformations would produce the same final image from the original preimage? Check your answer by performing the transformations. Then make a conjecture that generalizes your findings.



11. a. Graph triangle DEF with vertices $D(2, 6)$, $E(5, 6)$, and $F(5, 1)$ on the coordinate grid.
- b. Next graph triangle $D'E'F'$, the image of triangle DEF after a reflection across the y-axis.
- c. On the same coordinate grid, graph triangle $D''E''F''$, the image of triangle $D'E'F'$ after a translation of 7 units down and 2 units to the right.
- d. **Analyze Relationships** Find a different sequence of transformations that will transform triangle DEF to triangle $D''E''F''$.



LESSON 9.3 Properties of Rotations

 **FL** 8.G.1.1

Verify experimentally the properties of rotations...
Also 8.G.1.1a, 8.G.1.1b, 8.G.1.1c, 8.G.1.3



ESSENTIAL QUESTION

How do you describe the properties of rotation and their effect on the congruence and orientation of figures?

EXPLORE ACTIVITY 1

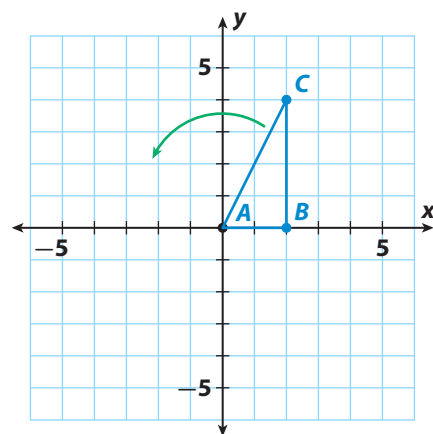
 **FL** 8.G.1.1

Exploring Rotations

A **rotation** is a transformation that turns a figure around a given point called the **center of rotation**. The image has the same size and shape as the preimage.

The triangle shown on the grid is the preimage. You will use the origin as the center of rotation.

- Trace triangle ABC onto a piece of paper. Cut out your traced triangle.
- Rotate your triangle 90° counterclockwise about the origin. The side of the triangle that lies along the x -axis should now lie along the y -axis.
- Sketch the image of the rotation. Label the images of points A , B , and C as A' , B' , and C' .
- Describe the motion modeled by the rotation.
Rotate _____ degrees _____ about the origin.
- Check that the motion you described in **D** is the same motion that maps point A onto A' , point B onto B' , and point C onto C' .



Reflect

- Communicate Mathematical Ideas** How are the size and the orientation of the triangle affected by the rotation?

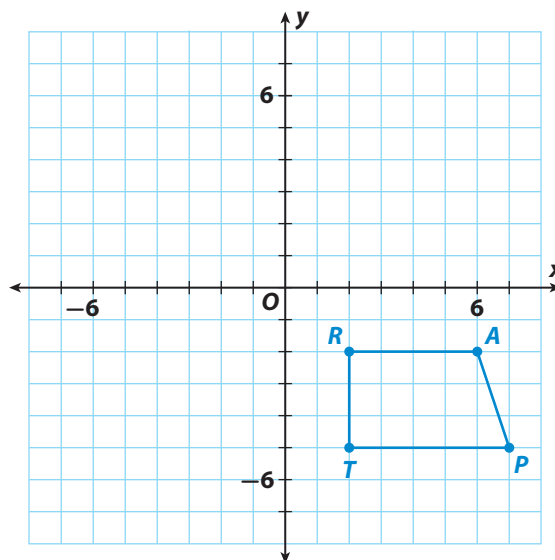
- Rotate triangle ABC 90° clockwise about the origin. Sketch the result on the coordinate grid above. Label the image vertices A'' , B'' , and C'' .



Properties of Rotations

Use trapezoid $TRAP$ to investigate the properties of rotations.

- A** Trace the trapezoid onto a piece of paper. Include the portion of the x - and y -axes bordering the third quadrant. Cut out your tracing.
- B** Place your trapezoid and axes on top of those in the figure. Then use the axes to help rotate your trapezoid 180° counterclockwise about the origin. Sketch the image of the rotation of your trapezoid in this new location. Label the vertices of the image T' , R' , A' , and P' .



- C** Use a ruler to measure the sides of trapezoid $TRAP$ in centimeters.

$$TR = \underline{\hspace{2cm}} \quad RA = \underline{\hspace{2cm}}$$

$$AP = \underline{\hspace{2cm}} \quad TP = \underline{\hspace{2cm}}$$

- D** Use a ruler to measure the sides of trapezoid $T'R'A'P'$ in centimeters.

$$T'R' = \underline{\hspace{2cm}} \quad R'A' = \underline{\hspace{2cm}}$$

$$A'P' = \underline{\hspace{2cm}} \quad T'P' = \underline{\hspace{2cm}}$$

- E** What do you notice about the lengths of corresponding sides of the two figures?

- F** Use a protractor to measure the angles of trapezoid $TRAP$.

$$m\angle T = \underline{\hspace{2cm}} \quad m\angle R = \underline{\hspace{2cm}} \quad m\angle A = \underline{\hspace{2cm}} \quad m\angle P = \underline{\hspace{2cm}}$$

- G** Use a protractor to measure the angles of trapezoid $T'R'A'P'$.

$$m\angle T' = \underline{\hspace{2cm}} \quad m\angle R' = \underline{\hspace{2cm}} \quad m\angle A' = \underline{\hspace{2cm}} \quad m\angle P' = \underline{\hspace{2cm}}$$

- H** What do you notice about the measures of corresponding angles of the two figures?

- I** Which sides of trapezoid $TRAP$ are parallel? _____

Which sides of trapezoid $T'R'A'P'$ are parallel? _____

What do you notice? _____

Reflect

3. **Make a Conjecture** Use your results from **E**, **H**, and **I** to make a conjecture about rotations.

4. Place your tracing back in its original position. Then perform a 180° clockwise rotation about the origin. Compare the result with the result of the transformation in **B**.

Graphing Rotations

To rotate a figure in the coordinate plane, rotate each of its vertices. Then connect the vertices to form the image.

EXAMPLE 1



FL 8.G.1.3

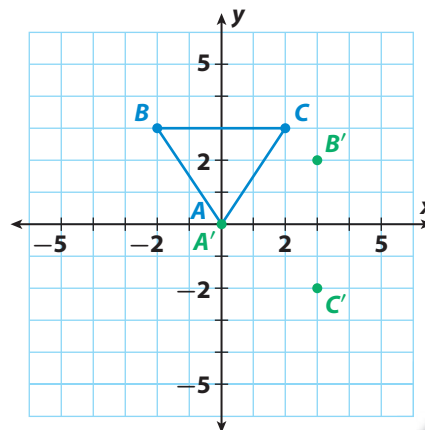
The figure shows triangle ABC . Graph the image of triangle ABC after a rotation of 90° clockwise.

STEP 1

Rotate the figure clockwise from the y -axis to the x -axis. Point A will still be at $(0, 0)$.

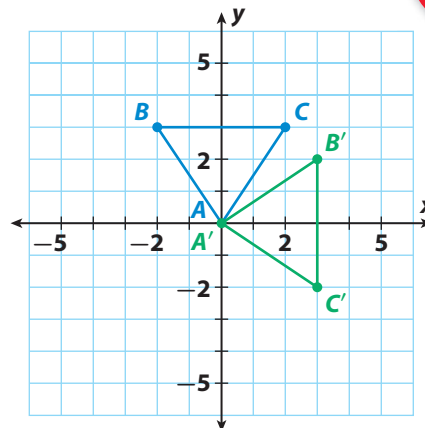
Point B is 2 units to the left of the y -axis, so point B' is 2 units above the x -axis.

Point C is 2 units to the right of the y -axis, so point C' is 2 units below the x -axis.



STEP 2

Connect A' , B' , and C' to form the image triangle $A'B'C'$.



Reflect

5. Is the image congruent to the preimage? How do you know?



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Math Talk

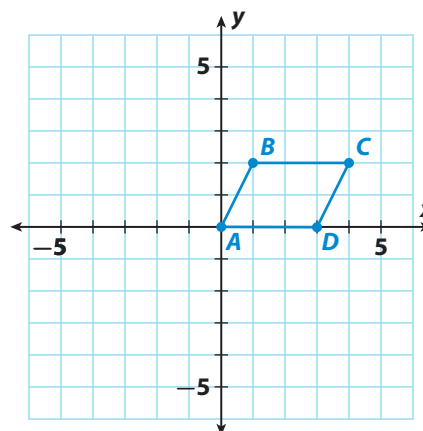
Mathematical Practices

How is the orientation of the triangle affected by the rotation?

YOUR TURN

Graph the image of quadrilateral $ABCD$ after each rotation.

6. 180°
7. 270° clockwise
8. Find the coordinates of Point C after a 90° counterclockwise rotation followed by a 180° rotation.



Guided Practice

1. **Vocabulary** A rotation is a transformation that turns a figure around a given _____ called the center of rotation.

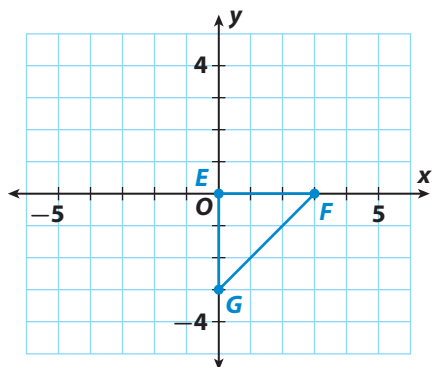
Siobhan rotates a right triangle 90° counterclockwise about the origin.

2. How does the orientation of the image of the triangle compare with the orientation of the preimage? ([Explore Activity 1](#))

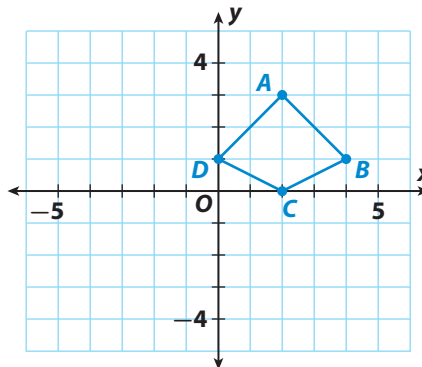
3. Is the image of the triangle congruent to the preimage? ([Explore Activity 2](#))

Draw the image of the figure after the given rotation about the origin. ([Example 1](#))

4. 90° counterclockwise



5. 180°



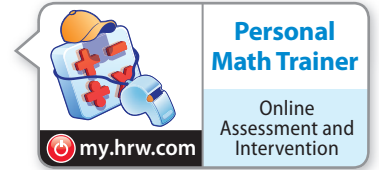
ESSENTIAL QUESTION CHECK-IN

6. What are the properties of rotations?

9.3 Independent Practice



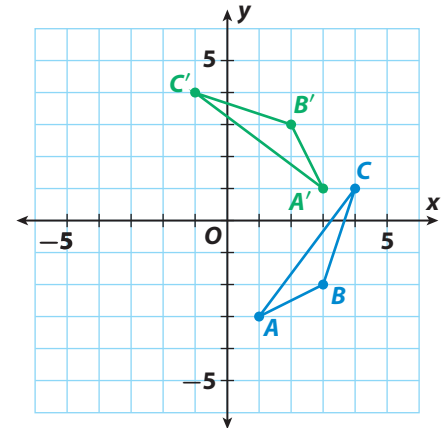
FL 8.G.1.1, 8.G.1.3



7. The figure shows triangle ABC and a rotation of the triangle about the origin.

a. How would you describe the rotation?

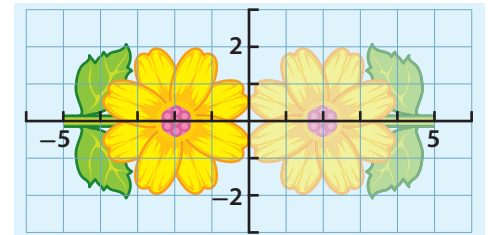
b. What are the coordinates of the image?



8. The graph shows a figure and its image after a transformation.

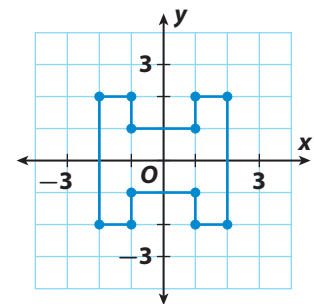
a. How would you describe this as a rotation?

b. Can you describe this as a transformation other than a rotation? Explain.



9. What type of rotation will preserve the orientation of the H-shaped figure in the grid?

10. A point with coordinates $(-2, -3)$ is rotated 90° clockwise about the origin. What are the coordinates of its image?

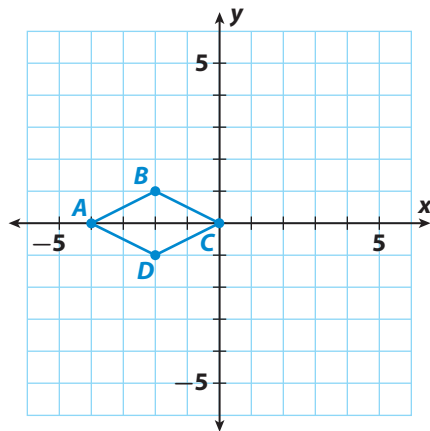


Complete the table with rotations of 180° or 90° . Include the direction of rotation for rotations of 90° .

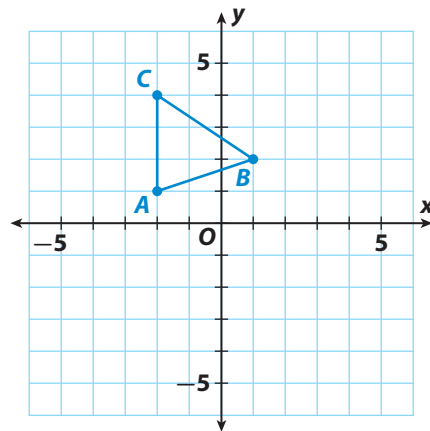
	Shape in quadrant	Image in quadrant	Rotation
11.	I	IV	
12.	III	I	
13.	IV	III	

Draw the image of the figure after the given rotation about the origin.

14. 180°



15. 270° counterclockwise



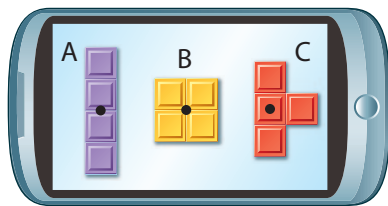
16. Is there a rotation for which the orientation of the image is always the same as that of the preimage? If so, what?

H.O.T.

FOCUS ON HIGHER ORDER THINKING

17. **Problem Solving** Lucas is playing a game where he has to rotate a figure for it to fit in an open space. Every time he clicks a button, the figure rotates 90° clockwise. How many times does he need to click the button so that each figure returns to its original orientation?

Figure A _____
Figure B _____
Figure C _____



18. **Make a Conjecture** Triangle ABC is reflected across the y -axis to form the image $A'B'C'$. Triangle $A'B'C'$ is then reflected across the x -axis to form the image $A''B''C''$. What type of rotation can be used to describe the relationship between triangle $A''B''C''$ and triangle ABC ?

19. **Communicate Mathematical Ideas** Point A is on the y -axis. Describe all possible locations of image A' for rotations of 90° , 180° , and 270° . Include the origin as a possible location for A .

Work Area

Algebraic Representations of Transformations



FL 8.G.1.3

Describe the effect of ..., translations, rotations, and reflections on two-dimensional figures using coordinates.



ESSENTIAL QUESTION

How can you describe the effect of a translation, rotation, or reflection on coordinates using an algebraic representation?

Algebraic Representations of Translations

The rules shown in the table describe how coordinates change when a figure is translated up, down, right, and left on the coordinate plane.

Translations	
Right a units	Add a to the x -coordinate: $(x, y) \rightarrow (x + a, y)$
Left a units	Subtract a from the x -coordinate: $(x, y) \rightarrow (x - a, y)$
Up b units	Add b to the y -coordinate: $(x, y) \rightarrow (x, y + b)$
Down b units	Subtract b from the y -coordinate: $(x, y) \rightarrow (x, y - b)$



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EXAMPLE 1



FL 8.G.1.3

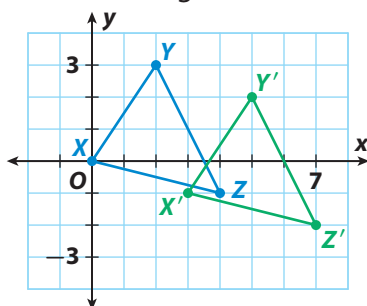
Triangle XYZ has vertices $X(0, 0)$, $Y(2, 3)$, and $Z(4, -1)$. Find the vertices of triangle $X'Y'Z'$ after a translation of 3 units to the right and 1 unit down. Then graph the triangle and its image.

Add 3 to the x -coordinate of each vertex and subtract 1 from the y -coordinate of each vertex.

STEP 1 Apply the rule to find the vertices of the image.

Vertices of $\triangle XYZ$	Rule: $(x + 3, y - 1)$	Vertices of $\triangle X'Y'Z'$
$X(0, 0)$	$(0 + 3, 0 - 1)$	$X'(3, -1)$
$Y(2, 3)$	$(2 + 3, 3 - 1)$	$Y'(5, 2)$
$Z(4, -1)$	$(4 + 3, -1 - 1)$	$Z'(7, -2)$

STEP 2 Graph triangle XYZ and its image.



Math Talk

Mathematical Practices

When you translate a figure to the left or right, which coordinate do you change?

YOUR TURN

1. A rectangle has vertices at $(0, -2)$, $(0, 3)$, $(3, -2)$, and $(3, 3)$. What are the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x - 6, y - 3)$? Describe the translation.

Algebraic Representations of Reflections

The signs of the coordinates of a figure change when the figure is reflected across the x -axis and y -axis. The table shows the rules for changing the signs of the coordinates after a reflection.

Reflections	
Across the x -axis	Multiply each y -coordinate by -1 : $(x, y) \rightarrow (x, -y)$
Across the y -axis	Multiply each x -coordinate by -1 : $(x, y) \rightarrow (-x, y)$

EXAMPLE 2



FL 8.G.1.3

My Notes

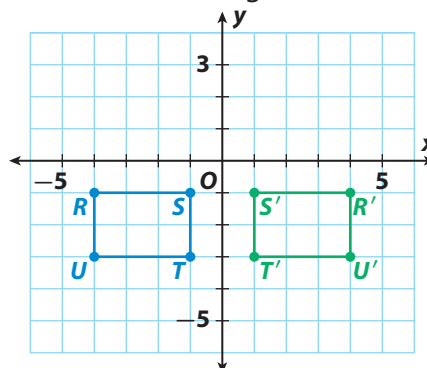
Rectangle $RSTU$ has vertices $R(-4, -1)$, $S(-1, -1)$, $T(-1, -3)$, and $U(-4, -3)$. Find the vertices of rectangle $R'S'T'U'$ after a reflection across the y -axis. Then graph the rectangle and its image.

STEP 1 Apply the rule to find the vertices of the image.

Multiply the x -coordinate of each vertex by -1 .

Vertices of $RSTU$	Rule: $(-1 \cdot x, y)$	Vertices of $R'S'T'U'$
$R(-4, -1)$	$(-1 \cdot (-4), -1)$	$R'(4, -1)$
$S(-1, -1)$	$(-1 \cdot (-1), -1)$	$S'(1, -1)$
$T(-1, -3)$	$(-1 \cdot (-1), -3)$	$T'(1, -3)$
$U(-4, -3)$	$(-1 \cdot (-4), -3)$	$U'(4, -3)$

STEP 2 Graph rectangle $RSTU$ and its image.



YOUR TURN

2. Triangle ABC has vertices $A(-2, 6)$, $B(0, 5)$, and $C(3, -1)$. Find the vertices of triangle $A'B'C'$ after a reflection across the x -axis.



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Algebraic Representations of Rotations

When points are rotated about the origin, the coordinates of the image can be found using the rules shown in the table.

Rotations	
90° clockwise	Multiply each x -coordinate by -1 ; then switch the x - and y -coordinates: $(x, y) \rightarrow (y, -x)$
90° counterclockwise	Multiply each y -coordinate by -1 ; then switch the x - and y -coordinates: $(x, y) \rightarrow (-y, x)$
180°	Multiply both coordinates by -1 : $(x, y) \rightarrow (-x, -y)$

EXAMPLE 3



FL 8.G.1.3

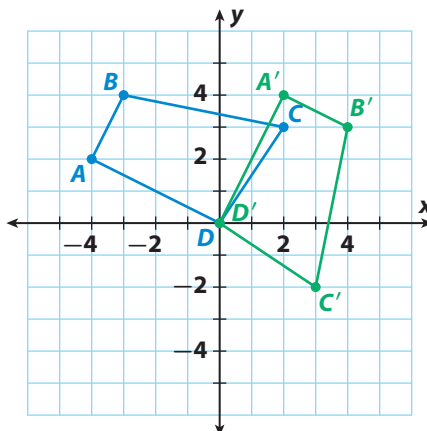
Quadrilateral $ABCD$ has vertices at $A(-4, 2)$, $B(-3, 4)$, $C(2, 3)$, and $D(0, 0)$. Find the vertices of quadrilateral $A'B'C'D'$ after a 90° clockwise rotation. Then graph the quadrilateral and its image.

STEP 1 Apply the rule to find the vertices of the image.

Multiply the x -coordinate of each vertex by -1 , and then switch the x - and y -coordinates.

Vertices of $ABCD$	Rule: $(y, -x)$	Vertices of $A'B'C'D'$
$A(-4, 2)$	$(2, -1 \cdot (-4))$	$A'(2, 4)$
$B(-3, 4)$	$(4, -1 \cdot (-3))$	$B'(4, 3)$
$C(2, 3)$	$(3, -1 \cdot 2)$	$C'(3, -2)$
$D(0, 0)$	$(0, -1 \cdot 0)$	$D'(0, 0)$

STEP 2 Graph the quadrilateral and its image.



Reflect

3. **Communicate Mathematical Ideas** How would you find the vertices of an image if a figure were rotated 270° clockwise? Explain.



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YOUR TURN

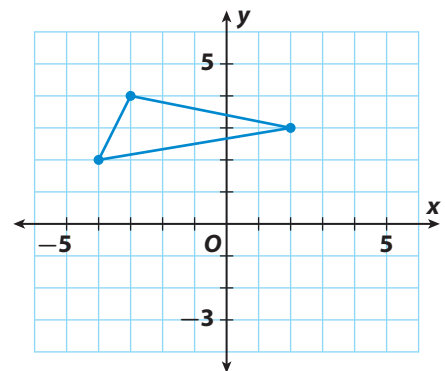
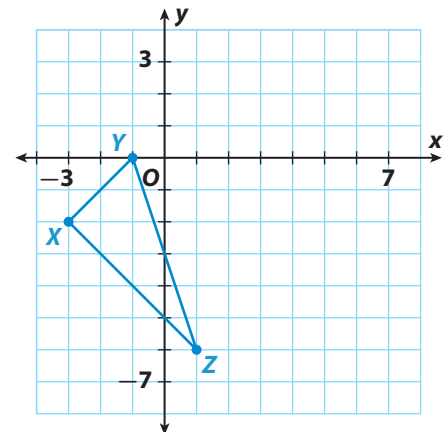
4. A triangle has vertices at $J(-2, -4)$, $K(1, 5)$, and $L(2, 2)$. What are the coordinates of the vertices of the image after the triangle is rotated 90° counterclockwise?

Guided Practice

1. Triangle XYZ has vertices $X(-3, -2)$, $Y(-1, 0)$, and $Z(1, -6)$. Find the vertices of triangle $X'Y'Z'$ after a translation of 6 units to the right. Then graph the triangle and its image. (Example 1)

2. Describe what happens to the x - and y -coordinates after a point is reflected across the x -axis. (Example 2)

3. Use the rule $(x, y) \rightarrow (y, -x)$ to graph the image of the triangle at right. Then describe the transformation. (Example 3)



ESSENTIAL QUESTION CHECK-IN

4. How do the x - and y -coordinates change when a figure is translated right a units and down b units?

9.4 Independent Practice



FL 8.G.1.3



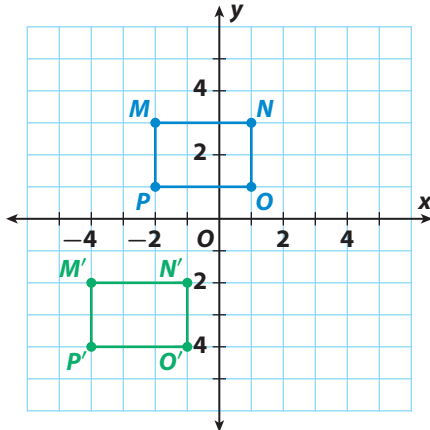
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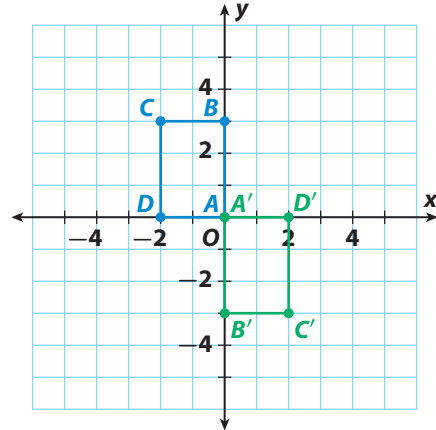
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Write an algebraic rule to describe each transformation.
Then describe the transformation.

5.



6.

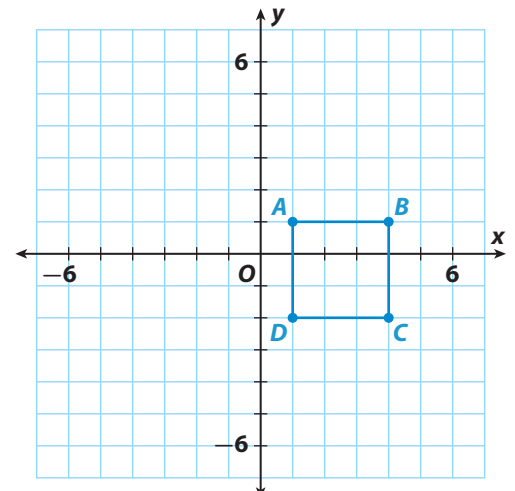


7. Triangle XYZ has vertices $X(6, -2.3)$, $Y(7.5, 5)$, and $Z(8, 4)$. When translated, X' has coordinates $(2.8, -1.3)$. Write a rule to describe this transformation. Then find the coordinates of Y' and Z' .

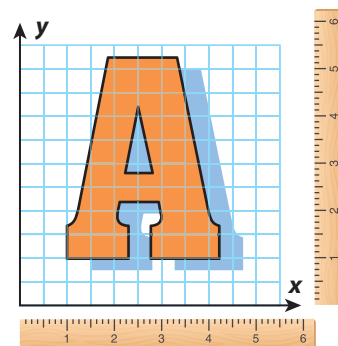
8. Point L has coordinates $(3, -5)$. The coordinates of point L' after a reflection are $(-3, -5)$. Without graphing, tell which axis point L was reflected across. Explain your answer.

9. Use the rule $(x, y) \rightarrow (x - 2, y - 4)$ to graph the image of the rectangle. Then describe the transformation.

10. Parallelogram $ABCD$ has vertices $A(-2, -5\frac{1}{2})$, $B(-4, -5\frac{1}{2})$, $C(-3, -2)$, and $D(-1, -2)$. Find the vertices of parallelogram $A'B'C'D'$ after a translation of $2\frac{1}{2}$ units down.



11. Alexandra drew the logo shown on half-inch graph paper. Write a rule that describes the translation Alexandra used to create the shadow on the letter A.



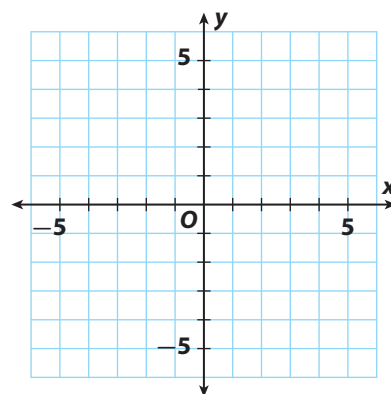
12. Kite $KLMN$ has vertices at $K(1, 3)$, $L(2, 4)$, $M(3, 3)$, and $N(2, 0)$. After the kite is rotated, K' has coordinates $(-3, 1)$. Describe the rotation, and include a rule in your description. Then find the coordinates of L' , M' , and N' .



FOCUS ON HIGHER ORDER THINKING

13. **Make a Conjecture** Graph the triangle with vertices $(-3, 4)$, $(3, 4)$, and $(-5, -5)$. Use the transformation (y, x) to graph its image.

- a. Which vertex of the image has the same coordinates as a vertex of the original figure? Explain why this is true.



- b. What is the equation of a line through the origin and this point?

- c. Describe the transformation of the triangle.

14. **Critical Thinking** Mitchell says the point $(0, 0)$ does not change when reflected across the x - or y -axis or when rotated about the origin. Do you agree with Mitchell? Explain why or why not.

15. **Analyze Relationships** Triangle ABC with vertices $A(-2, -2)$, $B(-3, 1)$, and $C(1, 1)$ is translated by $(x, y) \rightarrow (x - 1, y + 3)$. Then the image, triangle $A'B'C'$, is translated by $(x, y) \rightarrow (x + 4, y - 1)$, resulting in $A''B''C''$.

- a. Find the coordinates for the vertices of triangle $A''B''C''$.

- b. Write a rule for one translation that maps triangle ABC to triangle $A''B''C''$.

Work Area

9.5 Congruent Figures



FL 8.G.1.2

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.



ESSENTIAL QUESTION

What is the connection between transformations and figures that have the same shape and size?

EXPLORE ACTIVITY

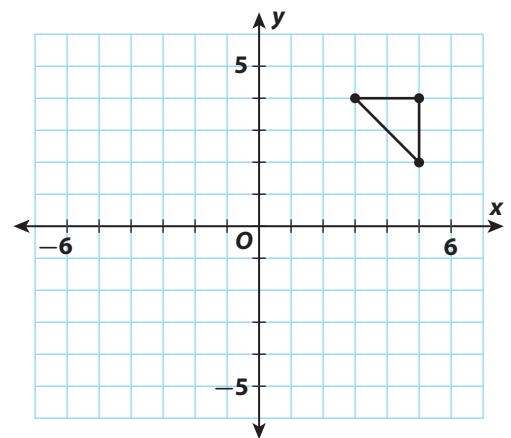


FL 8.G.1.2

Combining Transformations

Apply the indicated series of transformations to the triangle. Each transformation is applied to the image of the previous transformation, not the original figure. Label each image with the letter of the transformation applied.

- A** Reflection across the x-axis
- B** $(x, y) \rightarrow (x - 3, y)$
- C** Reflection across the y-axis
- D** $(x, y) \rightarrow (x, y + 4)$
- E** Rotation 90° clockwise around the origin
- F** Compare the size and shape of the final image to that of the original figure.



Reflect

1. Which transformation(s) change the orientation of figures? Which do not?
2. **Make a Conjecture** Two figures have the same size and shape. What does this indicate about the figures?



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Congruent Figures

Recall that segments and their images have the same length and angles and their images have the same measure under a translation, reflection, or rotation. Two figures are said to be **congruent** if one can be obtained from the other by a sequence of translations, reflections, and rotations. Congruent figures have the same size and shape.

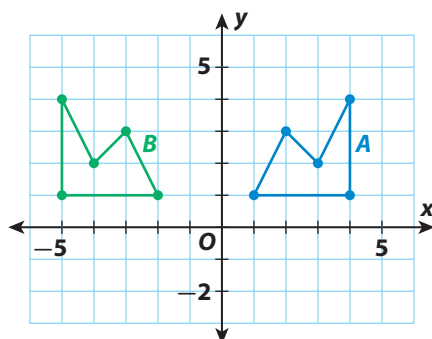
When you are told that two figures are congruent, there must be a sequence of translations, reflections, and/or rotations that transforms one into the other.

EXAMPLE 1



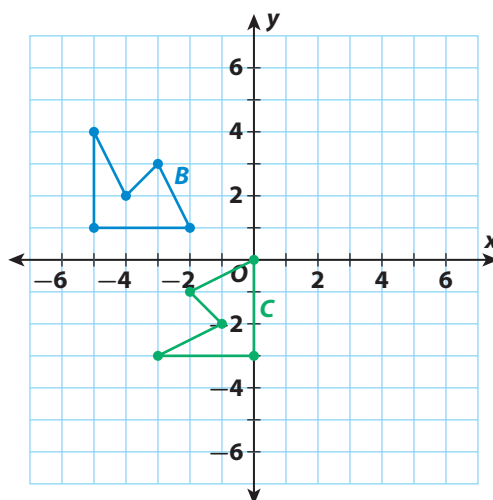
FL 8.G.1.2

- A** Identify a sequence of transformations that will transform figure A into figure B.



To transform figure A into figure B, you need to reflect it over the y-axis and translate one unit to the left. A sequence of transformations that will accomplish this is $(x, y) \rightarrow (-x, y)$ and $(x, y) \rightarrow (x - 1, y)$.

- B** Identify a sequence of transformations that will transform figure B into figure C.



Any sequence of transformations that changes figure B into figure C will need to include a rotation. A 90° counterclockwise rotation around the origin would result in the figure being oriented as figure C.

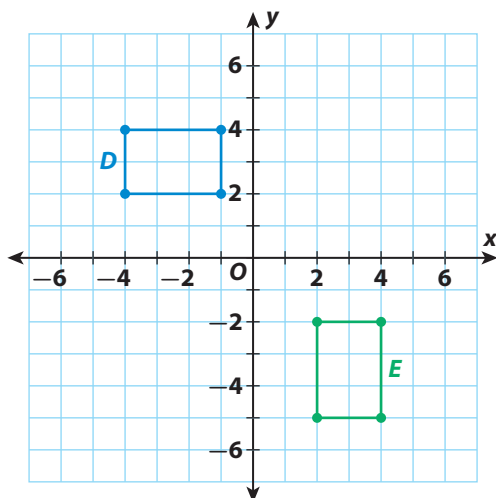
However, the rotated figure would be 2 units below and 1 unit to the left of where figure C is. You would need to translate the rotated figure up 2 units and right 1 unit.

Math Talk

How do you know that the sequence of transformations in Parts B and C must include a rotation?

The sequence of transformations is a 90° counterclockwise rotation about the origin, $(x, y) \rightarrow (-y, x)$, followed by $(x, y) \rightarrow (x + 1, y + 2)$.

- C** Identify a sequence of transformations that will transform figure D into figure E .



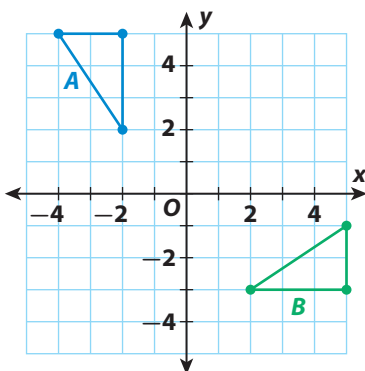
A sequence of transformations that changes figure D to figure E will need to include a rotation. A 90° clockwise rotation around the origin would result in the figure being oriented as figure E .

However, the rotated figure would be 6 units above where figure E is. You would need to translate the rotated figure down 6 units.

The sequence of transformations is a 90° clockwise rotation about the origin, $(x, y) \rightarrow (y, -x)$, followed by $(x, y) \rightarrow (x, y - 6)$.

YOUR TURN

3. Identify a sequence of transformations that will transform figure A into figure B .

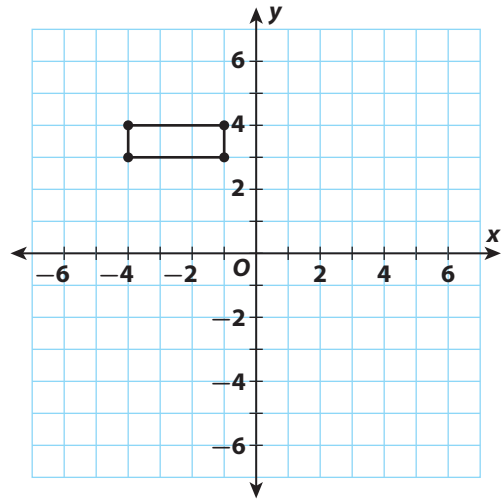


Guided Practice

1. Apply the indicated series of transformations to the rectangle. Each transformation is applied to the image of the previous transformation, not the original figure. Label each image with the letter of the transformation applied.

(Explore Activity)

- a. Reflection across the y -axis
- b. Rotation 90° clockwise around the origin
- c. $(x, y) \rightarrow (x - 2, y)$
- d. Rotation 90° counterclockwise around the origin
- e. $(x, y) \rightarrow (x - 7, y - 2)$



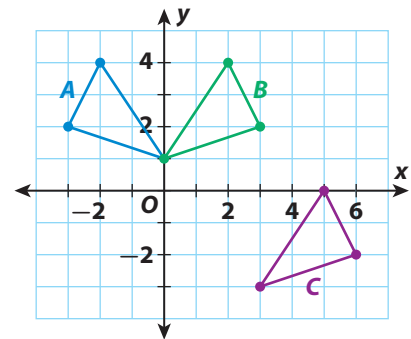
Identify a sequence of transformations that will transform figure A into figure C. (Example 1)

2. What transformation is used to transform figure A into figure B?

3. What transformation is used to transform figure B into figure C?

4. What sequence of transformations is used to transform figure A into figure C? Express the transformations algebraically.

5. **Vocabulary** What does it mean for two figures to be congruent?



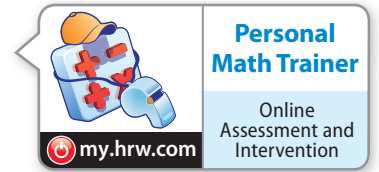
ESSENTIAL QUESTION CHECK-IN

6. After a sequence of translations, reflections, and rotations, what is true about the first figure and the final figure?

9.5 Independent Practice



FL 8.G.1.2



For each given figure *A*, graph figures *B* and *C* using the given sequence of transformations. State whether figures *A* and *C* have the same or different orientation.

7.

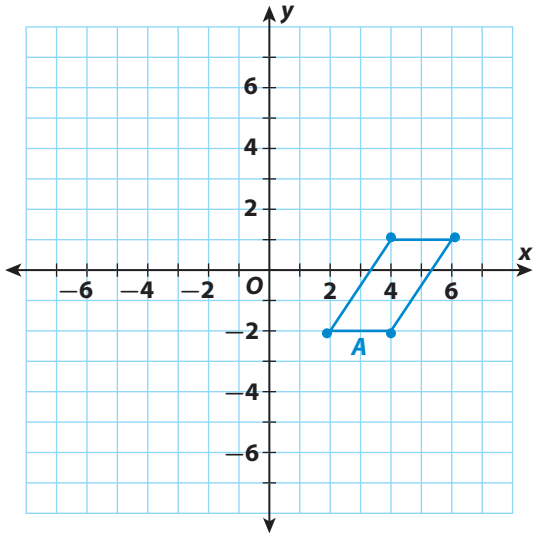


Figure *B*: a translation of 1 unit to the right and 3 units up

Figure *C*: a 90° clockwise rotation around the origin

8.

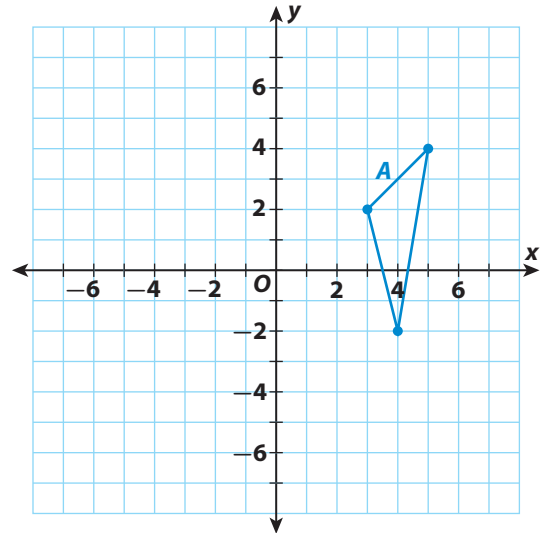


Figure *B*: a reflection across the *y*-axis

Figure *C*: a 180° rotation around the origin

9.

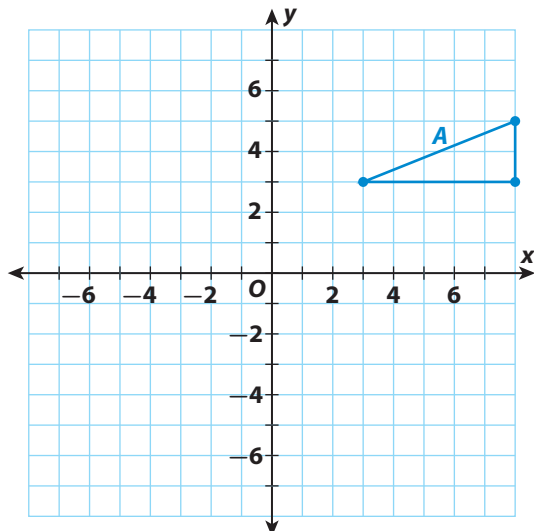


Figure *B*: a reflection across the *y*-axis

Figure *C*: a translation 2 units down

10.

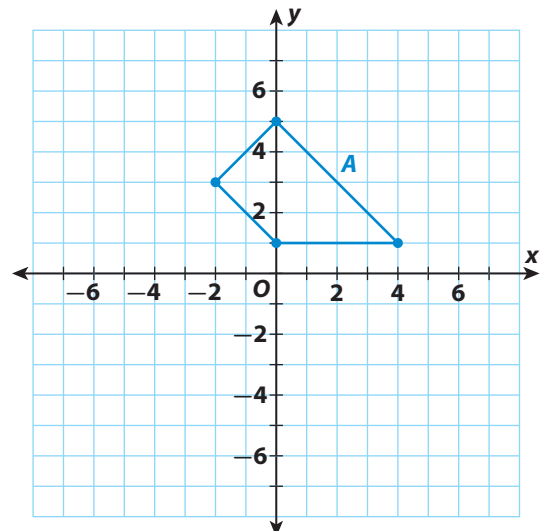
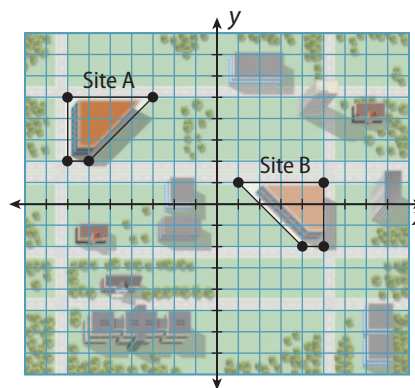


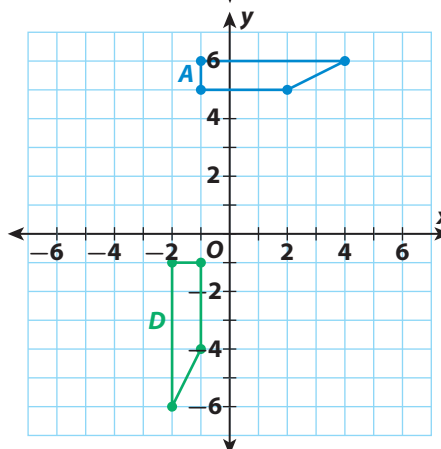
Figure *B*: a translation 2 units up

Figure *C*: a rotation of 180° around the origin

- 11. Represent Real-World Problems** A city planner wanted to place the new town library at site A. The mayor thought that it would be better at site B. What transformations were applied to the building at site A to relocate the building to site B? Did the mayor change the size or orientation of the library?



- 12. Persevere in Problem Solving** Find a sequence of three transformations that can be used to obtain figure D from figure A. Graph the figures B and C that are created by the transformations.



FOCUS ON HIGHER ORDER THINKING

- 13. Counterexamples** The Commutative Properties for Addition and Multiplication state that the order of two numbers being added or multiplied does not change the sum or product. Are translations and rotations commutative? If not, give a counterexample.

- 14. Multiple Representations** For each representation, describe a possible sequence of transformations.

a. $(x, y) \rightarrow (-x - 2, y + 1)$

b. $(x, y) \rightarrow (y, -x - 3)$

Work Area

Ready to Go On?



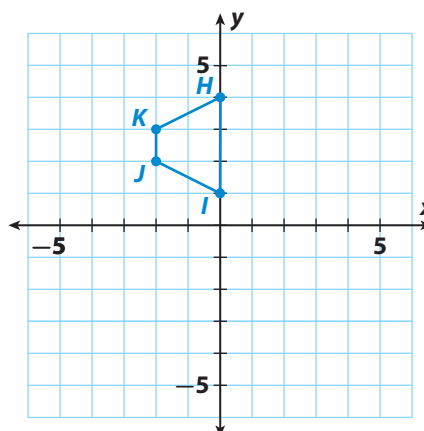
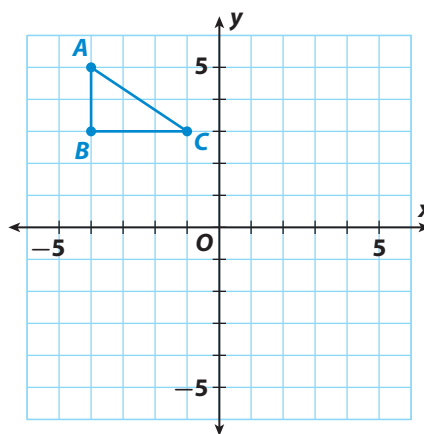
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9.1–9.3 Properties of Translations, Reflections, and Rotations

- Graph the image of triangle ABC after a translation of 6 units to the right and 4 units down. Label the vertices of the image A' , B' , and C' .
- On the same coordinate grid, graph the image of triangle ABC after a reflection across the x -axis. Label the vertices of the image A'' , B'' , and C'' .
- Graph the image of $HIJK$ after it is rotated 180° about the origin. Label the vertices of the image $H'I'J'K'$.



9.4 Algebraic Representations of Transformations

- A triangle has vertices at $(2, 3)$, $(-2, 2)$, and $(-3, 5)$. What are the coordinates of the vertices of the image after the translation $(x, y) \rightarrow (x + 4, y - 3)$?

9.5 Congruent Figures

- Vocabulary** Translations, reflections, and rotations produce a figure that is _____ to the original figure.
- Use the coordinate grid for Exercise 3. Reflect $H'I'J'K'$ over the y -axis, then rotate it 180° about the origin. Label the new figure $H''I''J''K''$.



ESSENTIAL QUESTION

- How can you use transformations to solve real-world problems?

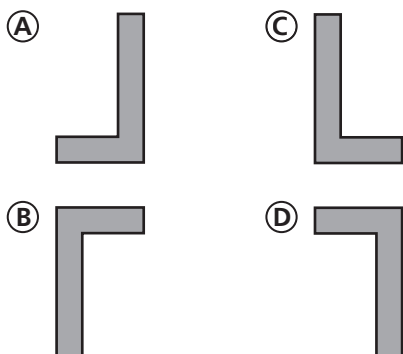
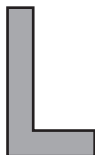


Assessment Readiness



Selected Response

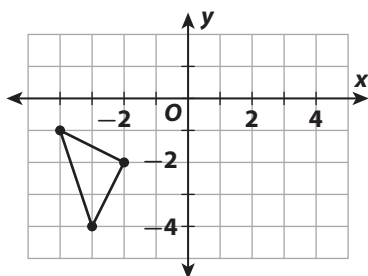
1. What would be the orientation of the figure L after a translation of 8 units to the right and 3 units up?



2. Figure A is reflected over the y-axis and then lowered 6 units. Which sequence describes these transformations?

- (A) $(x, y) \rightarrow (x, -y)$ and $(x, y) \rightarrow (x, y - 6)$
- (B) $(x, y) \rightarrow (-x, y)$ and $(x, y) \rightarrow (x, y - 6)$
- (C) $(x, y) \rightarrow (x, -y)$ and $(x, y) \rightarrow (x - 6, y)$
- (D) $(x, y) \rightarrow (-x, y)$ and $(x, y) \rightarrow (x - 6, y)$

3. What quadrant would the triangle be in after a rotation of 90° counterclockwise about the origin?



- (A) I (B) II (C) III (D) IV

4. Which rational number is greater than $-3\frac{1}{3}$ but less than $-\frac{4}{5}$?

- (A) -0.4 (C) -0.19
- (B) $-\frac{9}{7}$ (D) $-\frac{22}{5}$

5. Which of the following is **not** true of a trapezoid that has been reflected across the x-axis?
- (A) The new trapezoid is the same size as the original trapezoid.
- (B) The new trapezoid is the same shape as the original trapezoid.
- (C) The new trapezoid is in the same orientation as the original trapezoid.
- (D) The x-coordinates of the new trapezoid are the same as the x-coordinates of the original trapezoid.
6. A triangle with coordinates $(6, 4)$, $(2, -1)$, and $(-3, 5)$ is translated 4 units left and rotated 180° about the origin. What are the coordinates of its image?
- (A) $(2, 4)$, $(-2, -1)$, $(-7, 5)$
- (B) $(4, 6)$, $(-1, 2)$, $(5, -3)$
- (C) $(4, -2)$, $(-1, 2)$, $(5, 7)$
- (D) $(-2, -4)$, $(2, 1)$, $(7, -5)$

Mini-Task

7. A rectangle with vertices $(3, -2)$, $(3, -4)$, $(7, -2)$, $(7, -4)$ is reflected across the x-axis and then rotated 90° counterclockwise.

a. In what quadrant does the image lie?

b. What are the vertices of the image?

c. What other transformations produce the same image?
